A significant weakness of the equilibrium class of models is that they sometimes fail to describe the term structure of interest rates observed in the market. Traders find this shortcoming to be frustrating. Thus, a second class of models has won a much wider acceptance. These models start with the assertion that the current term structure of interest rates contains no arbitrage opportunities. These models are known as no-arbitrage models. It is important to note that as the term structure of interest rates changes over time, the no-arbitrage model parameters must be reestimated accordingly.

Ho and Lee (1986, 1990) developed the first no-arbitrage model. Hull and White (1990) present a very important no-arbitrage model that extended the Vasicek (1977) equilibrium model while maintaining relative ease of use. The no-arbitrage model that is probably used the most by market participants to price interest rate derivatives is the Heath–Jarrow–Morton (HJM) model, introduced in 1992. It can be shown that most other no-arbitrage models are special cases of the HJM model. An important distinction of the HJM model is that this approach is based on the evolution of forward interest rates (as opposed to spot interest rates). The HJM model allows for wide flexibility. Fortunately, software such as FinancialCAD is available to those wishing to use the no-arbitrage approach to value interest rate derivatives.

20.2.5 Options on Swaps

While trading volume in all derivative markets has grown dramatically in recent years, perhaps one of the most explosive growth areas has been in the market for options on swaps. Much of the growth is probably attributable to the increased recognition that these instruments provide considerable risk-shifting capability. In addition, these instruments, while sophisticated and apparently rather complex, are relatively simple.

An option to enter an interest rate swap at some later date is called a swaption.[17] That is, a swaption is based on a forward-start interest rate swap. For example, a six-month into a three-year swaption is an option to enter into a three-year interest rate swap six months from now. One key to understanding swaptions is that the right to enter the swap can be viewed in two ways:

a. Call swaptions. These swaptions are also called **payer swaptions**. The holder of a payer swaption has the right, but not the obligation, to enter the swap as the fixed rate *payer*. Here, holders of the swaption are looking to protect themselves against subsequent increases in the fixed rate. A likely candidate to purchase this type of swap is a firm that will be borrowing at a floating rate at some future date but wants to swap to a fixed rate. A call swaption is similar to a call option on an interest rate or a put on a debt instrument.

b. Put swaptions. These swaptions are also called **receiver swaptions**. The holder of a receiver swaption has the right, but not the obligation, to enter the swap as the fixed-rate *receiver*. Here, holders of the swaption want to protect themselves against subsequent decreases in the fixed rate. A firm would purchase this type of swaption if it currently holds a portfolio of floating-rate securities but is thinking about swapping to receive a fixed rate. A put swaption is similar to a put on an interest rate or a call on a debt instrument.

As with any option, the purchaser of a swaption must pay a premium to receive the rights conferred by the swaption. The swaption premium is expressed as basis points per dollar of notional principal. Various factors influence the value of a swaption. First, note that a swaption can have American-style or European-style exercise. The time to expiration of the swaption refers to the time from now until the time the swap can be entered. The fixed rate on the swaption is called the strike price. The underlying rate is the fair fixed rate on a forward-start interest rate swap. The volatility of this fair fixed rate is also required.

20.2.5.1 European Swaption Valuation

Because a swap can be viewed as a collection of forward contracts, a European option on a swap can be valued by Black's (1976) version of the BSOPM presented in Chapter 18.

A swaption has a strike rate K_x that is the fixed rate that will be swapped against the floating rate if the option is exercised. In a call swaption, or payer swaption, the buyer has the right to become the fixed-rate payer. The pricing model is

$$\text{call swaption} = (N \times B)[F_r N(d_1) - K_x N(d_2)] \tag{20.4}$$

In Equation (20.4), N represents the notional principal, and B represents the present value of a security that pays $1/n$ at all i payment dates of the swap. The underlying asset of the call is a T-year forward-start swap where payments are made every nth interval. Thus, if b_i represents the present value of one dollar received at time i, we write

$$B = \frac{1}{n}\sum_{i}^{nT} b_i(0,i) \tag{20.5}$$

Also, in Equation (20.04), F_r represents today's fair fixed rate on a forward swap, and:

$$d_1 = \frac{\ln(F_r/K_x) + (\sigma^2/2)T}{\sigma\sqrt{T}} \tag{20.6}$$

$$d_2 = \frac{\ln(F_r/K_x) - (\sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{20.7}$$

In a put swaption, or receiver swaption, the buyer has the right to receive the fixed rate. Using the notation of Equation (20.4), the value of a put swaption is given by:

$$\text{put swaption} = (N \times B)[K_x N(-d_2) - F_r N(-d_1)] \tag{20.8}$$

20.2.5.2 Pricing a Call Swaption: Example

Recall the example of a forward swap from Chapter 13 (Section 13.1.4). Assume that in that example, the forward swap will begin in 47 days. The notional principal is \$50 million. Assume a 360-day year. If the forward swap is executed today, both parties, the fixed-rate payer and the receive-fixed party, are obligated to the terms of the forward swap. That is, in any forward contract, including a forward swap, both parties have the right *and* the obligation to enter into the swap. An alternative for the fixed-rate payer would be to purchase a payer swaption, in which case the fixed-rate payer would have the right, but not the obligation, to enter the swap.

To begin the process of valuing a payer swaption, recall the Eurodollar futures settlement prices from Table 10.2. For example, on July 28, 1999, the September Eurodollar futures contract settled at 94.555, implying a borrowing rate of 5.445% from the expiration of the September contract to the expiration of the December contract, a period assumed here to be 90 days. Thus, the first swap payment would occur in 137 days. The value of b_1 is given by

$$b_1 = e^{-0.05445(137/365)} = 0.979770.$$

The other elements of b are computed in accordance with Table 20.1.

TABLE 20.1 Computing the Value of *b* for a Payer Swaption from Price Data Observed on July 28, 1999

Month	Settle Yield	Days Until First Swap Payment[1]	Unannualized Rate[2]	*b*
Sept. 1999	0.05445	137	0.020437397	0.979770
Dec. 1999	0.0581	228	0.036292603	0.964358
Mar. 2000	0.05815	320	0.050980822	0.950297
June 2000	0.0605	412	0.068290411	0.933989
Sept. 2000	0.06235	503	0.085923425	0.917664
Dec. 2000	0.06465	593	0.10503411	0.900294
Mar. 2001	0.0645	685	0.121047945	0.885991
June 2001	0.0651	777	0.13858274	0.870591

[1] Days between July 28, 1999 and the twelfth day of the following contract month. For example, there are 137 days between July 28, 1999 and December 12, 1999. There are 777 days between July 28, 1999 and September 12, 2001.
[2] Settle yield × (days until first swap payment/365).

Using these values for b_i, and noting that $n=4$ and $T=2$, the value for B as stated in equation (20.5) is:

$$B=\frac{1}{4}(0.979770+0.964358+0.950297+0.933989$$
$$+0.917664+0.900294+0.885991+0.870591)=1.85073$$

Suppose the strike price is set equal to the fair fixed rate on the forward swap, 6.09%. The time to maturity of the swaption equals the time when the swap would start (i.e., 47 days hence or 47/360 year). The final, and important input, is the volatility of the swap rate. Assume, for this example, that it is 20% per year, or 0.20. The values for d_1 and d_2 are calculated as follows[18]:

$$d_1=\frac{\ln(0.0609/0.0609)+(0.20^2/2)47/360}{0.20\sqrt{47/360}}=0.036132$$

$$d_2=d_1-\sigma\sqrt{T}=0.036132-0.20\sqrt{47/360}=-0.036132$$

From the cumulative normal distribution, $N(d_1)=N(0.036132)=0.514417$ and $N(d_2)=N(-0.036132)=0.485588$. Combining this information yields a call swaption value of

$$\begin{aligned}\text{call swaption} &= (N\times B)[F_rN(d_1)-K_rN(d_2)]\\ &=(50{,}000{,}000\times 1.85073)[0.0609\times 0.514417-0.0609\times 0.485588]\\ &=\$162{,}465,\ \text{or}\ \$0.0032493\ \text{per dollar of notional principal}\\ &\qquad\text{(i.e., 0.32493 basis point)}\end{aligned}$$

This is an example of how to price a European swaption. In practice, swaptions can be American or European, or an exotic combination of the two, which is known as a *Bermuda* option. For example, consider a swaption with a maturity of four years. It may be European during the first year and American thereafter. To value an American swaption or a Bermudan swaption, it is necessary to use lattice techniques that make normality assumptions (see Jarrow and Turnbull, 2000, Chapter 15) or a lattice technique that approximates the Heath–Jarrow–Morton model (the HJM model does not make normality or lognormality assumptions.) For more details see, Jarrow (1995) or Buetow and Fabozzi (2001). The next section discusses exotic options of other types.

20.3 Exotic Options

From about the late 1980s, options have evolved that allow hedgers to shift risks in ways that ordinary, traded options cannot accomplish. These options have become known as exotic options.

Exotic options are generally divided into two broad groups. One group is widely known as **path-dependent** options and the other group is often called **path-independent**, or **free-range**, options. We begin with a discussion of free-range options.[19]

20.3.1 Free-Range Exotic Options

Recall from Chapter 18 that the BSOPM formulas for calls and puts are

$$C = SN(d_1) - Ke^{-rT}N(d_2) \tag{18.3}$$

and

$$P = Ke^{-rT}N(-d_2) - SN(-d_1) \tag{18.8}$$

where we use the following definitions:

S = price of the underlying asset
K = strike price of the call option
r = risk-free interest rate
T = time to expiration
$N(d)$ = cumulative standard normal distribution function

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

σ = the standard deviation of the underlying asset's returns
$\ln(S/K)$ = the natural logarithm of S/K
e^{-rT} = the exponential function of $-rT$.

20.3.1.1 European Digital Options

Digital options, also known as binary options, are important to financial engineers as building blocks in the construction of more complex options. At expiration, a digital call option is worth $1 if $S_T > K$ and zero otherwise. For binary puts, the expiration value is \$1 if $K > S_T$ and zero otherwise. Thus, the payoff from digital call put options is discontinuous at the strike price.[20] Before expiration, the values of digital options (C_{digital} and P_{digital}) are given by:

$$C_{\text{digital}} = e^{-rT} N(d_2) \tag{20.9}$$

and

$$P_{\text{digital}} = e^{-rT} N(-d_2) \tag{20.10}$$

If the payoff of the digital option is multiplied by the strike price, the digital option is known as a strike-or-nothing (son) option. Thus, at expiration, a strike-or-nothing call option is worth \$$K$ if $S_T > K$ and zero otherwise. For strike-or-nothing puts, the expiration value is \$$K$ if $K > S_T$ and zero otherwise. As such, before expiration, the value of these options is:

$$C_{\text{son}} = Ke^{-rT} N(d_2) \tag{20.11}$$

and

$$P_{\text{son}} = Ke^{-rT} N(-d_2) \tag{20.12}$$

Asset-or-nothing (aon) options, a natural companion to strike-or-nothing options, are also useful to financial engineers. The payoff at expiration for an asset-or-nothing call is \$$S_T$ if $S_T > K$ and zero otherwise. For asset-or-nothing puts, the expiration value is \$$S_T$ if $K > S_T$ and zero otherwise. Before expiration, these options are valued as follows:

$$C_{\text{aon}} = SN(d_1) \tag{20.13}$$

and

$$P_{\text{aon}} = SN(-d_1) \tag{20.14}$$

Note that the formulas for asset-or-nothing and strike-or-nothing options are simply parts of the BSOPM. Thus, one can see that the call option value given by the BSOPM represents a portfolio comprising a long position in an asset-or-nothing call option and a short position in a strike-or-nothing call option. Also, a put option value given by the BSOPM represents a portfolio made up of a long position in a strike-or-nothing put option and a short position in an asset-or-nothing put option.

20.3.1.2 European Gap Options

Gap options also have a discontinuous payoff profile. However, unlike the payoff to the ordinary digital option, the payoff to a gap option is not flat on both sides of the strike price. The payoff to a gap call option is $S_T - G$ if $S_T > K$ and zero otherwise. Note that this is the same payoff as that of an ordinary call option if $G = K$. Thus, to make a gap call option different from an ordinary call, G must be greater than or less than K. Before expiration, the value of a gap option can be thought of as the value of an ordinary call plus an adjustment for the difference between K and G. That is, for a gap call, we write:

$$C_{\text{gap}} = \left[SN(d_1) - Ke^{-rT}N(d_2)\right] + (K - G)e^{-rT}N(d_2) \tag{20.15}$$

Equation (20.15) shows that the value of a gap call equals the value of a long position in an asset-or-nothing call when $G=0$.

The payoff to a gap put option is $G-S_T$ if $K>S_T$ and zero otherwise. Note that this is the same payoff as that of an ordinary put option if $G=K$. Again, to make a gap put option different from an ordinary put option, G must differ from K. Before expiration, the value of a gap put option can be thought of as the value of an ordinary put plus an adjustment for the difference between G and K. That is, a gap put is expressed thus:

$$P_{\text{gap}} = \left[Ke^{-rT}N(-d_2) - SN(-d_1)\right] + (G - K)e^{-rT}N(-d_2) \tag{20.16}$$

Equation (20.16) shows that the value of a gap put equals the value of a short position in an asset-or-nothing put when $G=0$.

20.3.1.3 European Paylater Options

Paylater options provide a means to protect the holder of the option against a disastrous move in the underlying asset—at zero initial cost. However, the premium of $\$L_c$ is paid if the option expires in the money, even if the option does not expire sufficiently in the money to pay the premium. Thus, the payoff at expiration for a paylater call option is S_T-K-L_c if $S_T>K$ and zero otherwise. Note that a paylater call option will have a negative payoff if $K<S_T<K+L_c$.

Because a paylater call option can be viewed as a combination of an ordinary call option less a payment of $\$L_c$ times the value of a digital call option, one can solve for the premium at initiation (i.e., at time 0) by stating

$$[C_{\text{paylater}}]_0 = \left[SN(d_1) - Ke^{-rT}N(d_2)\right]_0 - \left[L_c e^{-rT}N(d_2)\right]_0 \tag{20.17}$$

Because the value of a paylater call option is set to zero at time 0, the premium amount L_c is

$$0 = \left[SN(d_1) - Ke^{-rT}N(d_2)\right]_0 - \left[L_c e^{-rT}N(d_2)\right]_0$$

$$L_c = \frac{\left[SN(d_1) - Ke^{-rT}N(d_2)\right]_0}{\left[e^{-rT}N(d_2)\right]_0}$$

After time zero, say at arbitrary time t, the value of a paylater call option is

$$[C_{\text{paylater}}]_t = \left[SN(d_1) - Ke^{-rT}N(d_2)\right]_t - \left[L_c e^{-rT}N(d_2)\right]_t \tag{20.18}$$

Similarly, for puts,

$$0 = \left[Ke^{-rT}N(-d_2) - SN(-d_1)\right]_0 - \left[L_p e^{-rT}N(-d_2)\right]$$

$$L_p = \frac{\left[Ke^{-rT}N(-d_2) - SN(-d_1)\right]_0}{\left[e^{-rT}N(-d_2)\right]_0}$$

and

$$[P_{\text{paylater}}]_t = \left[Ke^{-rT}N(-d_2) - SN(-d_1)\right]_t - \left[L_p e^{-rT}N(-d_2)\right]_t \tag{20.19}$$

Although the holder of a paylater option does not have to pay for the option if it expires out of the money, the premium the holder must pay if the option finishes in the money is significant. For example, if $S = K = 100$, $r = 6\%$, $T = 90$ days, $\sigma = 30\%$, and there are no dividends, the BSOPM yields a call option value of about \$6.67. Equation (20.18) shows that a paylater call option premium is about \$13.29. Thus, if $S_T = 100.01$, the holder of the ordinary call option has lost \$6.66 while the cost to a paylater holder is \$13.28, about twice as much. In percentage terms, the difference is even higher for out-of-the-money options.

20.3.1.4 European Chooser Options

Chooser options are sometimes known as "as you like it" options or "options for the undecided" (Rubinstein, 1991). This is because at purchase, the chooser option is neither a call option nor a put option. The buyer of a chooser option has the right, at a prespecified time, to "choose" whether the option finishes its life as an ordinary call option or as an ordinary put option. Chooser options are a lower cost alternative to purchasing a straddle.[21]

Pricing a chooser option is surprisingly straightforward. First, let the chooser option expire at time T, but the holder of the chooser must "choose" at time t. The exercise price equals K. Of course, the holder of the chooser will convert the option into a call if, at time t, a call with strike equal K expiring at time T is more valuable than a put with the same strike and expiration date. That is, the holder will chose "call" if

$$C_t(S, T-t, K) > P_t(S, T-t, K) \tag{20.20}$$

By using a continuous time version of put–call parity, equation (20.20) can be written as follows:

$$C_t(S, T-t, K) > C_t(S, T-t, K) - S_t + Ke^{-r(T-t)} \tag{20.21}$$

Therefore, from Equation (20.21), you can see that the holder of a chooser option will chose call if, at time t, the stock price is greater than the discounted strike price.

At initiation, the payoffs to a chooser option can be replicated by the following portfolio:

A long position in an ordinary call option with a strike price equal to K and a time to maturity of T

A long position in an ordinary put with a strike price equal to $Ke^{-r(T-t)}$ and a time to maturity of t (the choice date)

20.3.1.5 Compound Options

A compound option is an option written on another option. There are four basic types of compound option: a call on a call, a put on a call, a call on a put, and a put on a put. A call option on a call is a cacall. A caput is a call on a put.[22]

Many securities and contracts can be modeled as compound options. Both Black and Scholes (1973) and Galai and Masulis (1976) point out that ordinary calls on 100 shares of stock are actually compound options. The owner of a call has an option on a firm's stock, but stock represents a call on the assets of a levered firm. When a firm's debt matures, the firm will either repay the principal due to the bondholders (if the firm's value is sufficient to repay this amount), leaving the residual amount for the stockholders, or default, in which case the bondholders take over the firm's assets and the stockholders get nothing (this will occur if the value of the firm's assets is less than the amount due to the bondholders). Thus, owning a call is a compound option on the underlying firm's assets.

Coupon-paying bonds and sinking fund bonds are compound options (Geske, 1977). For example, the stockholders of a firm that has issued a coupon bond own a stream of compound European options. At each coupon date, the firm has the option of defaulting or paying the bondholders off with the coupon in exchange for another European option that expires at the next coupon payment date. The underlying assets of each option are the firm's assets. When the stockholders pay the last coupon and principal, they then own the residual value of the firm's assets.

"Split-fee options" exist in the mortgage-backed securities market. These, too, are actually compound options in which the buyer initially buys the right (but not the obligation) to later buy an option to subsequently make or take delivery of bonds that are collateralized with mortgages.

20.3.1.6 Options on the Minimum or Maximum of Two Unknown Outcomes

Stulz (1982) introduced formulas for the pricing of options on the minimum or maximum of two risky assets. These options are exotic because, unlike ordinary options, there are two different underlying assets.[23]

There are call options on the maximum of two stochastic values, put options on the minimum of two stochastic values, call options on the minimum of two stochastic values, and put options on the maximum of two stochastic values.

20.3.2 Path-Dependent Options

Unlike the free-range options discussed earlier, one group of options has a value that depends on the path of the price of the underlying asset before option expiration. These exotic options are called path-dependent options.

20.3.2.1 European Barrier Options with One Barrier and No Dividends

Barrier options are now heavily traded in the over-the-counter markets. One appealing feature of these options is that they are less expensive than ordinary options. This is because the payoff to a barrier option depends on whether the price of the underlying asset reaches a critical level, known as the barrier, before option expiration. Intuitively, then, barrier options are less expensive than ordinary options because there are fewer positive payoff opportunities for barrier options.[24]

Call and put barrier options can both be divided into two groups. These two groups are "out-barrier" options (also known as knock-outs or outs) and "in-barrier" options (knock-ins or ins). A knock-out option ceases to exist when the value of the underlying asset touches the barrier level. By contrast, a knock-in option comes into existence when the underlying asset price reaches the barrier level. The name of a barrier option also depends on the price path of the underlying asset. Thus, an upward price path of the underlying asset leads to "up and in" and "up and out" barrier options, while a downward price path of the underlying asset leads to "down and out" and "down and in" barrier options. Thus, although there are eight basic categories of barrier options, we will present only the cases involving call options.

Barrier options sometimes contain a rebate feature. Under the terms of the rebate, the holder of an "out" option receives part of the premium paid for the option if the option is "knocked out." Similarly, the holder of an "in" option would receive a portion of the option premium if the option expires without being "knocked in."[25]

Merton (1973) and Reiner and Rubinstein (1991) have developed formulas used to price barrier options.[26] An interesting feature of barrier options is that in the absence of a rebate provision and given identical payoffs and barrier levels, the value of a European down-and-in barrier option plus the value of a European down-and-out barrier option equals the value of a call option value given by the BSOPM. That is,

$$C_{\text{BSOPM}} = C_{\text{down-out}} + C_{\text{down-in}} \tag{20.22}$$

The following pricing relationship also holds:

$$C_{\text{BSOPM}} = C_{\text{up-out}} + C_{\text{up-in}} \tag{20.23}$$

To proceed, add the following definitions to those in Section 20.3.1:

$$H = \text{barrier level}$$

$$w = \frac{\ln(S/H)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$y = \frac{\ln(H^2/SK)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$z = \frac{\ln(H/S)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$\mu = \frac{r - 0.5\sigma^2}{\sigma^2}$$

Given these definitions, if the barrier level H is less than the strike price K the value of a down-and-in call is given by:

$$C_{\text{down-in}} = S(H/S)^{2(\mu+1)}N(y) - Ke^{-rT}(H/S)^{2\mu}N(y - \sigma\sqrt{T}) \tag{20.24}$$

We can use Equations (20.22) and (20.24), to show that the value of a down-and-out call is given by $C_{\text{BSOPM}} - C_{\text{down-in}}$.[27]

If the barrier level H is greater than the strike price K, the value of an up-and-in call is given by

$$\begin{aligned} C_{\text{up-in}} &= SN(w) - Ke^{-rT}N(w - \sigma\sqrt{T}) \\ &+ S(H/S)^{2(\mu+1)}[N(z) - N(y)] \\ &- Ke^{-rT}(H/S)^{2\mu}[N(z - \sigma\sqrt{T}) - N(y - \sigma\sqrt{T})] \end{aligned} \tag{20.25}$$

and the value of an up-and-out call is computed by using Equations (20.23) and (20.25). Note that when the barrier level is less than or equal to the strike price, the value of an up-and-out call is zero because this barrier option cannot expire with a positive intrinsic value. Consequently, when the barrier level is less than or equal to the strike price, the value of an up-and-in call equals the value of a Black–Scholes call.

20.3.2.2 Lookback Options

The payoff to a lookback option depends on the value of the extreme stock price during the life of the option. Sometimes these options are called "no regrets" options. This is because, for example, the holder of a lookback call can buy the underlying asset at its lowest price between option initiation and option expiration.

There are several types of lookback option. A standard lookback call option yields proceeds at exercise equal to the stock price at exercise less the minimum stock price observed during the life of the option. These standard lookback options are sometimes known as "floating-strike" options. By contrast, a fixed-strike lookback call option gives proceeds at exercise equal to the difference between the highest stock price observed during the life of the option and the exercise price. These options are also known as "extreme" lookback options.

Goldman, Sosin, and Gatto (1979) developed a formula for pricing floating-strike, or standard lookback, options. For a floating-strike lookback call, their formula is

$$\begin{aligned} C_{\text{floating-strike}} &= SN(g) - S_{\min}e^{-rT}N(g - \sigma\sqrt{T}) \\ &+ Se^{-rT}\frac{\sigma^2}{2r}\left[\left(\frac{S}{S_{\min}}\right)^{-2r/\sigma^2} N\left(-g + \frac{2r}{\sigma}\sqrt{T}\right) - e^{rT}N(-g)\right] \end{aligned} \tag{20.26}$$

where

$$g = \frac{\ln(S/S_{\min}) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}.$$

Conze and Viswanathan (1991) developed formulas for extreme lookback options. For example, in cases of strike prices less than or equal to the maximum price achieved by the underlying asset, their formula for call options is

$$C_{\text{fixed-strike}} = SN(f) - S_{\max}e^{-rT}N(f-\sigma\sqrt{T}) + e^{-rT}(S_{\max}-K)$$
$$+ Se^{-rT}\frac{\sigma^2}{2r}\left[-\left(\frac{S}{S_{\max}}\right)^{-2r/\sigma^2} N\left(f-\frac{2r}{\sigma}\sqrt{T}\right) + e^{rT}N(f)\right] \qquad (20.27)$$

where

$$f = \frac{\ln(S/S_{\max}) + (r+\sigma^2/2)T}{\sigma\sqrt{T}}$$

While we have presented formulas only for European lookback call options, formulas exist for European lookback put options (see, e.g., Haug, 1998). In addition, there are many variants of lookback options. For example, "partial-time" lookback options exist. The unique feature of these options is as follows. For a partial-time, floating-strike lookback option, the end of the lookback period occurs before option expiration. For a partial-time fixed-strike lookback option, the beginning of the lookback period starts a predetermined time after option initiation. Also, there are **Russian** options, which are like an infinite-life lookback option. After initiation, the holder of a Russian option can wait as long as desired before exercising the option. Of course, upon exercising during the finite lookback period, the option ceases to exist.

20.3.2.3 Average Options

Average price options are quite popular and are used in many different over-the-counter markets. The payoff to average options, also known as "Asian" options, depends on the average price of the underlying asset during the life of the options.

There are two basic types of average option, average price and average strike. At expiration, an average price call option pays the maximum of zero or $S_{avg}-K$. The payoff to an average price put option is the maximum of zero or $K-S_{avg}$. At expiration, an average strike call pays the maximum of zero or S_T-S_{avg}. Accordingly, the expiration payoff to an average strike put is the maximum of zero or $S_{avg}-S_T$.

Average price options are cheaper than ordinary call options. This is because volatility is dampened when averaging the prices of the underlying asset. Generally, a simple arithmetic average is used to calculate the average price. In this case, however, it is not possible to use a formula to price these options. Thus, these options must be priced by means of analytical approximations or with computer simulations.[28]

20.3.3 Shouts and Ladders

Shouts and ladders can best be described by means of a short example. Consider the case of an option holder who has an in-the-money option that still has some time left to maturity. The option holder would like to guarantee a profit on this position. For traded options, the option holder could use a stop-loss order. However, for over-the-counter options, stop-loss orders may be unavailable. Shout options provide a substitute. A **shout option** gives the option holder the right to capture the intrinsic value portion of the option premium before expiration and keep the time value.

For example, suppose a three-month, one-shout call option exists on Intel Corporation. At the time the option was purchased, $S = K = \$75$. After one month, Intel stock stands at \$100. The option holder may "shout" and lock in the \$25 intrinsic value. The option holder still holds a two-month call option, but the strike price is adjusted to \$100.

If a shout option has numerous shouting opportunities, it is known as a **ladder**.

20.4 Summary

In this chapter, we present three currently important risk management topics: value at risk (VaR), credit derivatives, and exotic options.

VaR is an attempt to condense into a single figure an estimate of the price risk possessed by a portfolio of derivatives and other financial assets of a single firm. The price risk number obtained from a VaR model summarizes risk exposure into a dollar figure that purportedly represents the estimated maximum loss over an interval of time. As yet, there is no standard method to compute VaR. However, several accepted methods for computing VaR have emerged. These methods are the variance–covariance approach, the historical simulation method, and the Monte Carlo simulation method. For any VaR calculation method, it is important for the risk manager to assess the valuation impact of worst-case scenarios, such as the market crash of 1987.

The second major topic presented in this chapter is credit risk. Credit risk permeates market economies. In its most basic form, credit risk is the chance that a bond issuer will not make every coupon payment, that is, the chance that the bond issuer will default. Because default on a contract can result in material losses, financial institutions look to protect themselves against credit risk. There are many different types of credit derivative, and more are being invented each year. However, the two most popular credit derivatives are total return swaps and credit swaps.

The third topic in this chapter is exotic options. From about the late 1980s, options have evolved that allow hedgers to shift risks in ways that ordinary, traded options cannot accomplish. These options have become known as exotic options. Exotic options are generally divided into two broad groups. One group of exotic options is known as path-independent, or free-range options. Options in this group include digitals, gaps, and paylater, chooser, and compound options. The other group is widely known as path-dependent options. Examples of path-dependent options include barrier, lookback, and average options.

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